PRECALCULUS I/MATH 126 (2550)

SHANNON MYERS

- ~~100~~ 105 POINTS POSSIBLE
- YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED
- YOU MAY USE A SCIENTIFIC AND/OR A TI-83/84/85/86 CALCULATOR
- PROVIDE EXACT ANSWERS UNLESS OTHERWISE INDICATED

ONCE YOU BEGIN THE EXAM, YOU MAY NOT LEAVE THE PROCTORING CENTER UNTIL YOU ARE FINISHED...THIS MEANS NO BATHROOM BREAKS!

NAME Key

PRECALCULUS/MATH 126 (2550)/MYERS NAME Key

PLEASE MAKE SURE YOU ARE TAKING THE EXAM FOR THE CORRECT INSTRUCTOR AND CLASS!!!

EXAM 1/~~100~~ 105 POINTS POSSIBLE

CREDIT WILL BE AWARDED BASED ON WORK SHOWN. THERE WILL BE NO CREDIT FOR GUESSING. PLEASE PRESENT YOUR WORK IN AN ORGANIZED, EASY TO READ FASHION.

1. (6 POINTS) Let $g(x) = (x-3)^2 + 1$.

   a. (3 POINTS) Find the average rate of change from -2 to 1.

$g(-2) = [(-2)-3]^2 + 1$

$g(-2) = (-5)^2 + 1$

$g(-2) = 26$

$g(1) = [(1)-3]^2 + 1$

$g(1) = (-2)^2 + 1$

$g(1) = 5$

Average rate of change $= \dfrac{g(1) - g(-2)}{1-(-2)} = \dfrac{5-26}{3} = -21/3 = \boxed{-7}$

   b. (3 POINTS) Find an equation of the secant line containing $(-2, g(-2))$ and $(1, g(1))$. Give your result in the **point-slope form of the line**.

m_{sec} = average rat of change = -7

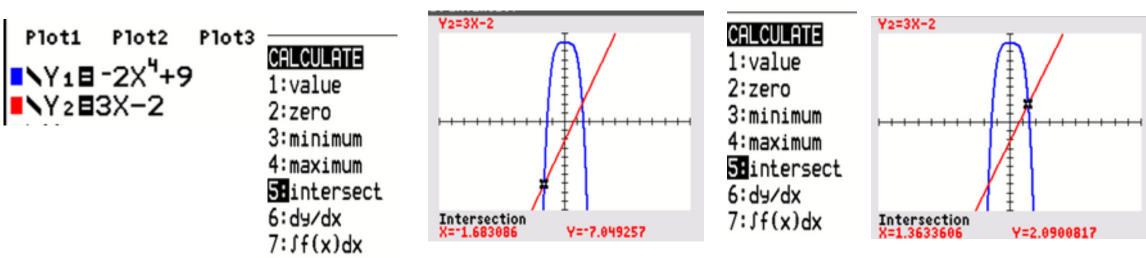
$(1,5)$: $y - y_1 = m(x - x_1)$ $\boxed{y - 5 = -7(x-1)}$

or

$(-2, 26)$: $y - y_1 = m(x - x_1)$ $y - 26 = -7(x-(-2))$ $\boxed{y - 26 = -7(x+2)}$

2. (6 POINTS) Use a graphing calculator to approximate the real solutions, if any, of the given equation rounded to **two decimal places**. All solutions lie between -10 and 10.

$-2x^4 + 9 = 3x - 2$



$\boxed{\{-1.68, 1.36\}}$

3. (8 POINTS) The function below is defined by three equations. Find the indicated function values.

$$f(x) = \begin{cases} -x-4 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 1-x^2 & \text{if } x \geq 1 \end{cases}$$

$f(1) = 3$

$f(4) = 1-(4)^2 = 1-16 = -15$

$f(0) = -(0)-4 = -4$

   a. $f(1) =$ $\boxed{3}$

   b. $f(4) =$ $\boxed{-15}$

   c. $f(0) =$ $\boxed{-4}$

   d. $2f(1) + f(0) =$ $2(3) + (-4) = \boxed{2}$

4. (8 POINTS) Find the difference quotient of f ; that is, find $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$, for the following function.

$f(x)=\frac{1}{x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \left[\frac{(1)}{(x+h)} \cdot \frac{x}{x} - \frac{(1)(x+h)}{(x)(x+h)}\right] \frac{1}{h}$$

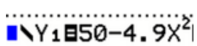
common denominator

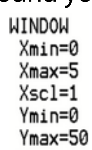
$$= \frac{x-(x+h)}{hx(x+h)}$$

$$= \frac{-\cancel{h}^{1}}{\cancel{h}_{1} x(x+h)} = \boxed{\frac{-1}{x(x+h)}}$$

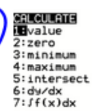
5. (9 POINTS) If a rock falls from a height of 50 meters on Earth, the height H in meters after x seconds is approximately $H(x)=50-4.9x^2$. Round your answers to **three decimal places**. Give the appropriate **units** with your answers.

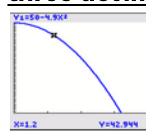
a and c



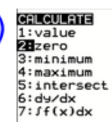


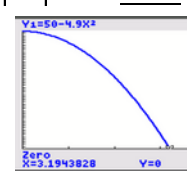
a)





c)

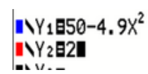


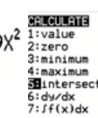


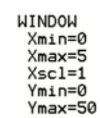
a. What is the height of the rock when $x=1.2$ seconds? 42.944 meters

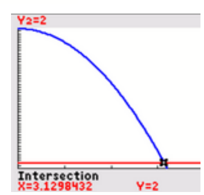
b. When is the height of the rock 2 meters? 3.130 seconds

b)





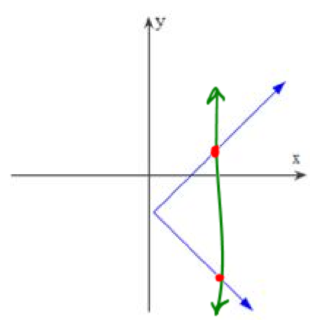




c. When does the rock hit the ground? 3.194 seconds

6. (1 POINTS) Use the vertical line test to determine if y is a function of x in the given graph.
Is y a function of x? Circle one response:

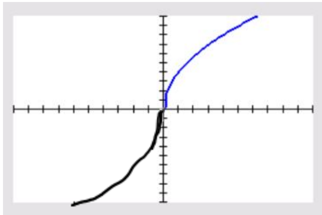
Yes No



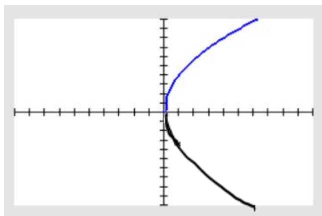
Fails since it passes through more than one point

7. (6 POINTS) Complete the graph so that the graph is symmetric with respect to the:

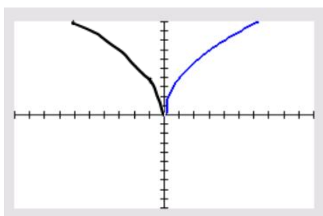
   a. Origin



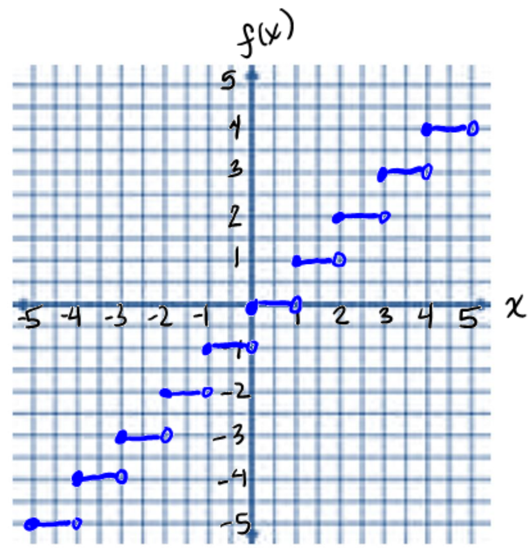
   b. *x*-axis



   c. *y*-axis



8. (6 POINTS) The function f is defined as follows: $f(x) = \text{int}(x)$.



   a. (3 POINTS) Graph the function. Be sure to label axes and scale.

   b. (2 POINTS)What is the domain? $(-\infty, \infty)$

   c. (1 POINTS)Is f continuous on its domain? NO

(discontinuity at every integer)

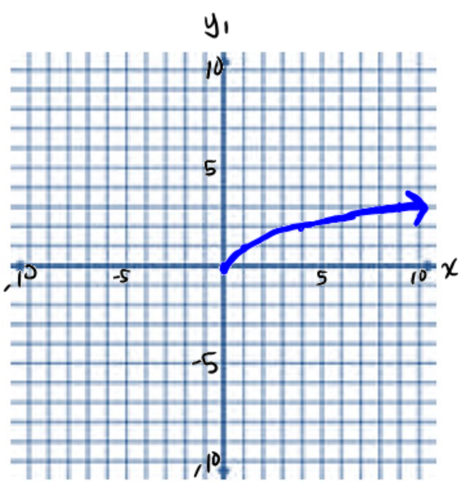
9. (4 POINTS) Give the domain of $f(x) = \dfrac{x-1}{x^2-1}$ in interval notation.

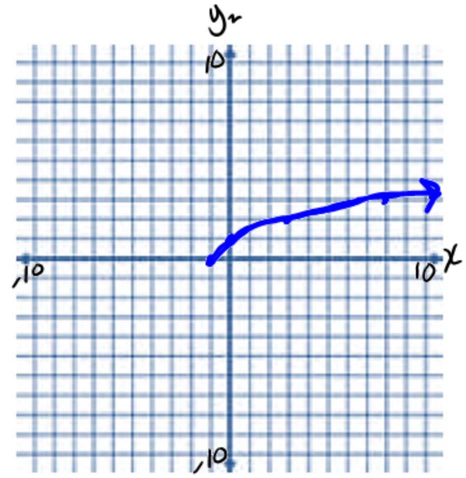
$$f(x) = \frac{x-1}{x^2-1} \rightarrow f(x) = \frac{\cancel{x-1}}{(x+1)\cancel{(x-1)}} \rightarrow f(x) = \frac{1}{x+1}, \quad x \neq 1$$

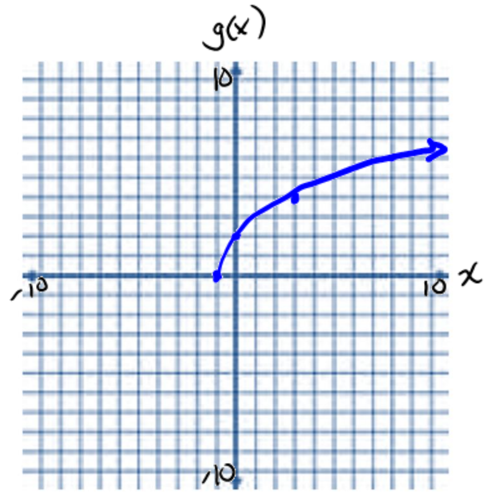
So $x \neq 1$ and $x + 1 \neq 0 \rightarrow x \neq -1$

$$\boxed{(-\infty, -1) \cup (-1, 1) \cup (1, \infty)}$$

10. (9 POINTS) Graph $g(x) = 2\sqrt{x+1}$ **by hand using transformations**. Fill in the blanks below to indicate the first two graphs.







$y_1 =$ $\sqrt{x}$

$y_2 =$ $\sqrt{x+1}$

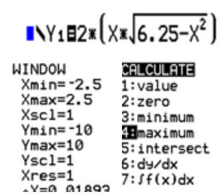
$g(x) = 2\sqrt{x+1}$

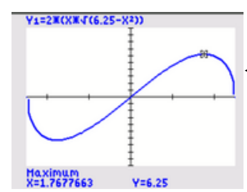
11. (10 POINTS) A rectangle is inscribed in a circle of radius 2.5 cm. Let $P = (x, y)$ be the point in quadrant I that is a vertex of the rectangle and is on the circle.

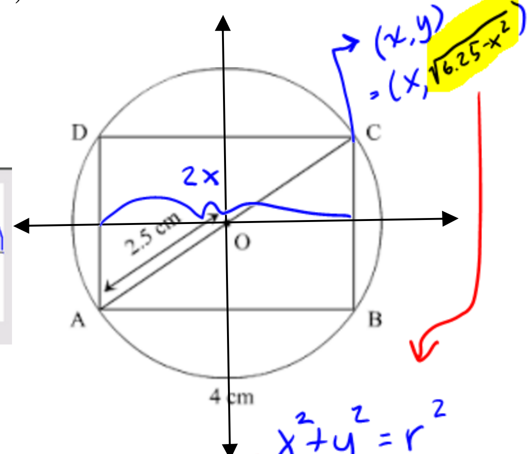
a. (4 POINTS) Express the area A of the rectangle as a function of *x*.

$A(x,y) = 2xy$

$\boxed{A(x) = 2x\sqrt{6.25 - x^2}}$







$x^2 + y^2 = r^2$

$x^2 + y^2 = (2.5)^2$

equation of circle w/radius 2.5.

$x^2 + y^2 = 6.25$

$y^2 = 6.25 - x^2$

$y = \pm\sqrt{6.25 - x^2}$

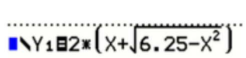
b. (1 POINT) Graph $A = A(x)$. For what value of *x* is A largest? 1.768 cm

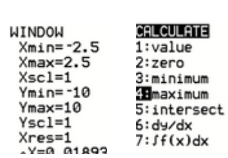
c. (4 POINTS) Express the perimeter p of the rectangle as a function of *x*.

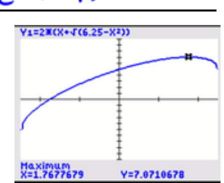
$p(x,y) = 2[2x + 2y] = 4(x + y)$

$\boxed{p(x) = 4(x + \sqrt{6.25 - x^2})}$

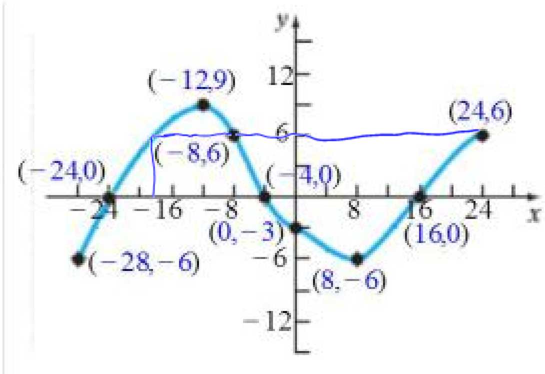
d. (1 POINT) Graph $p = p(x)$. For what value of *x* is p largest? $x \approx 1.768$ cm







12. (~~14~~ 18 POINTS) Consider the graph of $f(x)$ below.



a. For what value(s) of x is $f(x)$ equal to 0? $x = -24, -4, 16$

b. $f(0) =$ -3

c. On what interval(s) is f increasing? $(-28,-12) \cup (8,24)$

d. On what interval(s) is f decreasing? $(-12, 8)$

e. What is the domain of f ? $[-28, 24]$

f. What is the range of f ? $[-6, 9]$

g. For what values of x is $f(x) \geq 6$? $[-19,-8] \cup \{24\}$

h. What is the absolute minimum? -6

i. What is the absolute maximum? 9

13. (2 POINTS) Write the function for $f(x) = x^3$ which has been shifted 2 units up and 6 units to the left.

$\boxed{g(x) = (x+6)^3 + 2}$

14. (2 POINTS) If $(-1, 2)$ is a point on the graph of $y = f(x)$, what point of the following must be on the graph of $y = 3f(x)$?

$\boxed{(-1, 6)}$

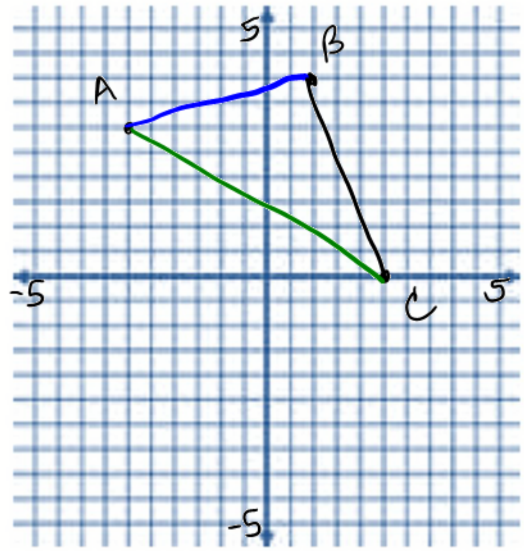
15. (7 POINTS) Determine if the triangle with vertices $\overset{A}{(-3,3)}$, $\overset{B}{(1,4)}$, and $\overset{C}{(2,0)}$ is a right triangle.

$m_{\overline{AB}} = \frac{4-3}{1-(-3)} = \frac{1}{4}$

$m_{\overline{AC}} = \frac{0-3}{2-(-3)} = -\frac{3}{5}$

$m_{\overline{BC}} = \frac{0-4}{2-1} = -4$

since one pair of slopes has a product of -1 (are negative reciprocals of each other), ΔABC is a right Δ.



OR

$d_{\overline{AB}} = \sqrt{(1-(-3))^2+(4-3)^2}$
$= \sqrt{17}$ units

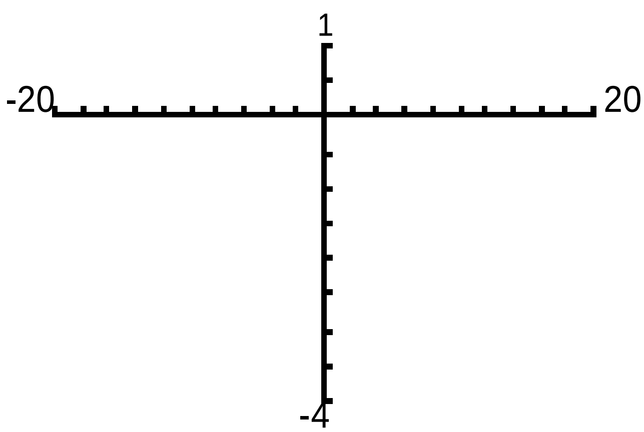
$d_{\overline{AC}} = \sqrt{(2-(-3))^2+(0-3)^2}$
$= \sqrt{34}$ units

$d_{\overline{BC}} = \sqrt{(2-1)^2+(0-4)^2}$
$= \sqrt{17}$ units

Now:
$(\sqrt{17})^2 + (\sqrt{17})^2 \overset{?}{=} \sqrt{34}$
$17 + 17 \overset{?}{=} 34$
$34 = 34$ ✓

yes ΔABC is a right Δ.

16. (3 POINTS) Determine the viewing window used.



a. Xmin = -20
b. Xmax = 20
c. Xscl = 2

d. Ymin = -4
e. Ymax = 1
f. Yscl = $1/2$